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Charge-dependence of the πNN coupling constant and charge-dependence of the NN interaction*

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Abstract. The recent determination of the charged πNN coupling constant, $g_{\pi^\pm}$, by the Uppsala Neutron Research Group implies that there may be considerable charge-splitting of the pion coupling constant. We investigate the consequences of this for the charge-independence breaking (CIB) of the $^1S_0$ scattering length, Δa_{CIB}. We find that Δa_{CIB} depends sensitively on the difference between $g_{\pi^\pm}$ and the neutral πNN coupling constant, g_{π^0}. Moreover, if $g^2_{\pi^\pm}$ is only about 3% larger than $g^2_{\pi^0}$, then the established theoretical explanation of Δa_{CIB} (in terms of pion mass splitting) is completely wiped out.

1 Introduction

From 1973 to 1987, there was a consensus that the πNN coupling constant is $g^2_\pi/4\pi = 14.3 \pm 0.2$ (equivalent to $f^2_\pi = 0.079 \pm 0.001$ [1]). This value was obtained by Bugg *et al.* [3] from the analysis of $\pi^\pm p$ data in 1973, and confirmed by Koch and Pietarinen [4] in 1980. Around that same time, the neutral-pion coupling constant was determined by Kroll [5] from the analysis of pp data by means of forward dispersion relations; he obtained $g^2_{\pi^0}/4\pi = 14.52 \pm 0.40$ (equivalent to $f^2_{\pi^0} = 0.080 \pm 0.002$).

The picture changed in 1987, when the Nijmegen group [6] determined the neutral-pion coupling constant in a partial-wave analysis of pp data and obtained $g^2_{\pi^0}/4\pi = 13.1 \pm 0.1$. Including also the magnetic moment interaction between protons in the analysis, the value shifted to 13.55 ± 0.13 in 1990 [7]. Triggered by these events, Arndt *et al.* [8] reanalysed the $\pi^\pm p$ data to determine the charged-pion coupling constant and obtained $g^2_{\pi^\pm}/4\pi = 13.31 \pm 0.27$.

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In subsequent work, the Nijmegen group also analysed np, $\bar{p}p$, and πN data. The status of their work as of 1993 is summarized in Ref. [11] where they claim that the most accurate values are obtained in their combined pp and np analysis yielding $g^2_{\pi^0}/4\pi = 13.47 \pm 0.11$ (equivalent to $f^2_{\pi^0} = 0.0745 \pm 0.0006$) and $g^2_{\pi^\pm}/4\pi = 13.54 \pm 0.05$ (equivalent to $f^2_{\pi^\pm} = 0.0748 \pm 0.0003$). The latest analysis of all $\pi^\pm p$ data below 2.1 GeV conducted by the VPI group using fixed-t and forward dispersion relation constraints has generated $g^2_{\pi^\pm}/4\pi = 13.75 \pm 0.15$ [12]. The VPI NN analysis extracted $g^2_{\pi^0}/4\pi \approx 13.3$ and $g^2_{\pi^\pm}/4\pi \approx 13.9$ as well as the charge-independent value $g^2_\pi/4\pi \approx 13.7$ [13, 14].

Also Bugg and coworkers have performed new determinations of the πNN coupling constant. Based upon precise $\pi^\pm p$ data in the 100–310 MeV range and applying fixed-t dispersion relations, they obtained the value $g^2_{\pi^\pm}/4\pi = 13.96 \pm 0.25$ (equivalent to $f^2_{\pi^\pm} = 0.0771 \pm 0.0014$) [15]. From the analysis of NN elastic data between 210 and 800 MeV, Bugg and Machleidt [16] have deduced $g^2_{\pi^\pm}/4\pi = 13.69 \pm 0.39$ and $g^2_{\pi^0}/4\pi = 13.94 \pm 0.24$.

Thus, it may appear that recent determinations show a consistent trend towards a lower value for g_π with no indication for substantial charge dependence.

Unfortunately this is not true. There is one recent determination that does not follow the current trend. Using the Chew extrapolation procedure, the Uppsala Neutron Research Group has deduced the charged-pion coupling constant from high precision np charge-exchange data at 162 MeV [17]. Their latest result is $g^2_{\pi^\pm}/4\pi = 14.52 \pm 0.26$ [18]. We note that the method used by the Uppsala Group is controversial [14, 19].

If one tries to summarize the confusing current picture then one may state that recent determinations of the neutral-pion coupling constant are, indeed, consistently on the low side with a value of $g^2_{\pi^0}/4\pi = 13.6 \pm 0.3$ covering about the range of current determinations.

However, there is no such consistent picture for the charged-pion coupling constant with recent determinations being up to nine standard deviations apart. If we trust the Uppsala result of $g^2_{\pi^\pm}/4\pi = 14.52 \pm 0.26$, then large charge-splitting of g_π exists.

This is the motive for the present paper in which we will investigate the impact of charge-splitting of g_π on our established theoretical understanding of the charge dependence of the nuclear force. In particular, we will look into the charge-independence breaking (CIB) of the $^1S_0$ scattering length, Δa_{CIB}. We find that Δa_{CIB} depends sensitively on the difference between $g_{\pi^\pm}$ and g_{π^0}. Moreover, if $g_{\pi^\pm}$ is only moderately larger than g_{π^0}, the established theoretical explanation of Δa_{CIB} (in terms of pion mass splitting) is completely wiped out.

2 Conventional explanation of the charge-dependence of the NN interaction

The equality between proton-proton (pp) [or neutron-neutron (nn)] and neutron-proton (np) nuclear interactions is known as charge independence—a symmetry that is slightly broken. This is seen most clearly in the $^1S_0$ nucleon-nucleon scattering lengths. The latest empirical values for the singlet scattering length a

and effective range r are [20, 21]:

$$\begin{aligned} a^N_{pp} &= -17.3 \pm 0.4 \text{ fm}, & r^N_{pp} &= 2.85 \pm 0.04 \text{ fm}, \\ a^N_{nn} &= -18.8 \pm 0.3 \text{ fm}, & r^N_{nn} &= 2.75 \pm 0.11 \text{ fm}, \\ a_{np} &= -23.75 \pm 0.01 \text{ fm}, & r_{np} &= 2.75 \pm 0.05 \text{ fm}. \end{aligned} \tag{1}$$

The values given for pp and nn scattering refer to the nuclear part of the interaction as indicated by the superscript N. Electromagnetic effects have been removed from the experimental values, which is model dependent. The uncertainties quoted for a^N_{pp} and r^N_{pp} are due to this model dependence.

It is useful to define the following averages:

$$\bar{a} \equiv \frac{1}{2}(a^N_{pp} + a^N_{nn}) \quad = \quad -18.05 \pm 0.5 \text{ fm}, \tag{2}$$

$$\bar{r} \equiv \frac{1}{2}(r^N_{pp} + r^N_{nn}) \quad = \quad 2.80 \pm 0.12 \text{ fm}. \tag{3}$$

By definition, charge-independence breaking (CIB) is the difference between the average of pp and nn, on the one hand, and np on the other:

$$\Delta a_{CIB} \equiv \bar{a} - a_{np} \quad = \quad 5.7 \pm 0.5 \text{ fm}, \tag{4}$$

$$\Delta r_{CIB} \equiv \bar{r} - r_{np} \quad = \quad 0.05 \pm 0.13 \text{ fm}. \tag{5}$$

Thus, the NN singlet scattering length shows a clear signature of CIB in strong interactions.

The current understanding is that the charge dependence of nuclear forces is due to differences in the up and down quark masses and electromagnetic interactions. On a more phenomenological level, major causes of CIB are the mass splittings of isovector mesons (particularly, π and ρ) and irreducible pion-photon exchanges.

It has been known for a long time that the difference between the charged and neutral pion masses in the one-pion-exchange (OPE) potential accounts for about 50% of Δa_{CIB}. Based upon the Bonn meson-exchange model for the NN interaction [22], also multiple pion exchanges have been taken into account. Including these interactions, about 80% of the empirical Δa_{CIB} can be explained [23, 24]. Ericson and Miller [25] obtained a very similar result using the meson-exchange model of Partovi and Lomon [26].

The CIB effect from OPE can be understood as follows. In nonrelativistic approximation [27] and disregarding isospin factors, OPE is given by

$$V_{1\pi}(g_\pi, m_\pi) = -\frac{g_\pi^2}{4M^2} \frac{(\boldsymbol{\sigma}_1 \cdot \mathbf{k})(\boldsymbol{\sigma}_2 \cdot \mathbf{k})}{m_\pi^2 + \mathbf{k}^2} \left(\frac{\Lambda^2 - m_\pi^2}{\Lambda^2 + \mathbf{k}^2} \right)^n \tag{6}$$

with M the average nucleon mass, m_π the pion mass, and $\mathbf{k}$ the momentum transfer. The above expression includes a form factor with cutoff mass Λ and exponent n.

For $S = 0$ and $T = 1$, where S and T denote the total spin and isospin of the two-nucleon system, respectively, we have

$$V^{01}_{1\pi}(g_\pi, m_\pi) = \frac{g_\pi^2}{m_\pi^2 + \mathbf{k}^2} \frac{\mathbf{k}^2}{4M^2} \left(\frac{\Lambda^2 - m_\pi^2}{\Lambda^2 + \mathbf{k}^2} \right)^n , \tag{7}$$

Table 1. Predictions for Δa_{CIB} as defined in Eq. (4) in units of fm without and with the assumption of charge-dependence of g_π.

	No charge-dependence of g_π		**Charge-dependent g_π:** $g^2_{\pi^0}/4\pi = 13.6$ $g^2_{\pi^\pm}/4\pi = 14.4$
	Ericson & Miller [25]	Li & Machleidt [24]	
1π	3.50	3.24	–1.58
2π	0.88	0.36	–1.94
$\pi\rho, \pi\sigma, \pi\omega$	—	1.04	–0.97
Sum	4.38	4.64	–4.49
Empirical		5.7 ± 0.5	

where the superscripts 01 refer to ST. In the $^1S_0$ state, this potential expression is repulsive. The charge-dependent OPE is then,

$$V^{01}_{1\pi}(pp) = V^{01}_{1\pi}(g_{\pi^0}, m_{\pi^0}) \tag{8}$$

for pp scattering, and

$$V^{01}_{1\pi}(np) = 2V^{01}_{1\pi}(g_{\pi^\pm}, m_{\pi^\pm}) - V^{01}_{1\pi}(g_{\pi^0}, m_{\pi^0}) \tag{9}$$

for np scattering.

If we assume charge-independence of g_π (i. e., $g_{\pi^0} = g_{\pi^\pm}$), then all CIB comes from the charge splitting of the pion mass, which is [28]

$$\begin{aligned} m_{\pi^0} &= 134.976\text{MeV}, && (10)\\ m_{\pi^\pm} &= 139.570\text{MeV}. && (11) \end{aligned}$$

Since the pion mass appears in the denominator of OPE, the smaller π^0-mass exchanged in pp scattering generates a larger (repulsive) potential in the $^1S_0$ state as compared to np where also the larger $\pi^\pm$-mass is involved. Moreover, the π^0-exchange in np scattering carries a negative sign, which further weakens the np OPE potential. The bottom line is that the pp potential is more repulsive than the np potential. The quantitative effect on Δa_{CIB} is about 3 fm (cf. Table 1).

We now turn to the CIB created by the 2π exchange (TPE) contribution to the NN interaction. There are many diagrams that contribute (see Ref. [24] for a complete overview). For our qualitative discussion here, we pick the largest of all 2π diagrams, namely, the box diagrams with $N\Delta$ intermediate states, Fig. 1. Disregarding isospin factors and using some drastic approximations [27], the amplitude for such a diagram is

$$V_{2\pi}(g_\pi, m_\pi) = -\frac{g^4_\pi}{16M^4}\frac{72}{25}\int \frac{d^3p}{(2\pi)^3}\frac{[\boldsymbol{\sigma}\cdot\mathbf{k}\boldsymbol{S}\cdot\mathbf{k}]^2}{(m^2_\pi+\mathbf{k}^2)^2(E_p+E^\Delta_p-2E_q)}\left(\frac{\Lambda^2-m^2_\pi}{\Lambda^2+\mathbf{k}^2}\right)^{2n}, \tag{12}$$

where $\mathbf{k} = \mathbf{p} - \mathbf{q}$ with $\mathbf{q}$ the relative momentum in the initial and final state (for simplicity, we are considering a diagonal matrix element); $E_p = \sqrt{M^2 + \mathbf{p}^2}$ and $E^\Delta_p = \sqrt{M^2_\Delta + \mathbf{p}^2}$ with $M_\Delta = 1232$ MeV the Δ-isobar mass; $\mathbf{S}$ is the spin

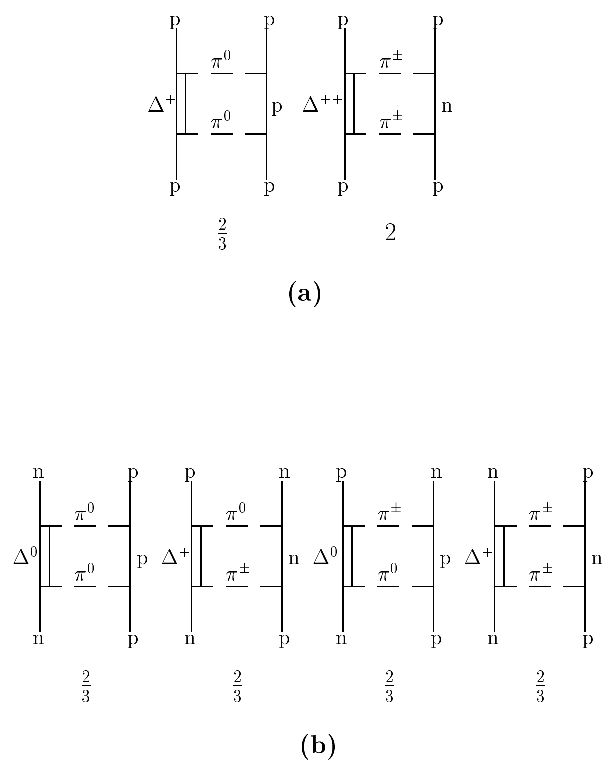

Figure 1. 2π-exchange box diagrams with $N\Delta$ intermediate states that contribute to (a) pp and (b) np scattering. The numbers below the diagrams are the isospin factors.

transition operator between nucleon and Δ. For the $\pi N\Delta$ coupling constant, $f_{\pi N\Delta}$, the quark-model relationship $f^2_{\pi N\Delta} = \frac{72}{25} f^2_{\pi NN}$ is used [22].

For small momentum transfers $\mathbf{k}$, this attractive contribution is roughly proportional to m_π^{-4}. Thus for TPE, the heavier pions will provide less attraction than the lighter ones. Charged and neutral pion exchanges occur for pp as well as for np, and it is important to take the isospin factors carried by the various diagrams into account. They are given in Fig. 1 below each diagram. For pp scattering, the diagram with double $\pi^\pm$ exchange carries the largest factor, while double $\pi^\pm$ exchange carries only a small relative weight in np scattering. Consequently, pp scattering is less attractive than np scattering which leads to an increase of Δa_{CIB} by 0.79 fm due to the diagrams of Fig. 1. The crossed diagrams of this type reduce this result and including all 2π exchange diagrams one finds a total effect of 0.36 fm [24]. Diagrams that go beyond 2π have also been investigated and contribute another 1 fm (see Table 1 for a summary).

In this way, pion-mass splitting explains about 80% of Δa_{CIB}.

3 Charge-dependence of the pion coupling constant and charge-dependence of the singlet scattering length

In this section, we will consider also charge-splitting of g_π, besides pion mass splitting.

As discussed in the Introduction, some current determinations of g_π may suggest the values

$$g^2_{\pi^0}/4\pi = 13.6 \,, \tag{13}$$
$$g^2_{\pi^\pm}/4\pi = 14.4 \,. \tag{14}$$

Accidentally, this splitting is—in relative terms—about the same as the pion-mass splitting; that is

$$\frac{g_{\pi^0}}{m_{\pi^0}} \approx \frac{g_{\pi^\pm}}{m_{\pi^\pm}} \,. \tag{15}$$

From the discussion in the previous section, we know that (for zero momentum transfer)

$$\text{OPE} \sim \left(\frac{g_\pi}{m_\pi}\right)^2 \tag{16}$$

and

$$\text{TPE} \sim \left(\frac{g_\pi}{m_\pi}\right)^4 \,, \tag{17}$$

which is not unexpected, anyhow. On the level of this qualitative discussion, we can then predict that any pionic charge-splitting satisfying Eq. (15) will create no CIB from pion exchanges. Consequently, a charge-splitting of g_π as given in Eqs. (13) and (14) will wipe out our established explanation of CIB of the NN interaction.

We have also conducted accurate numerical calculations based upon the Bonn meson-exchange model for the NN interaction [22]. The details of these calculations are spelled out in Ref. [24] where, however, no charge-splitting of g_π was

considered. Assuming the g_π of Eqs. (13) and (14), we obtain the Δa_{CIB} predictions given in the last column of Table 1. It is seen that the results of an accurate calculation go even beyond what the qualitative estimate suggested: the conventional CIB prediction is not only reduced, it is reversed. This is easily understood if one recalls that the pion mass appears in the propagator $(m_\pi^2 + \mathbf{k}^2)^{-1}$. Assuming an average $\mathbf{k}^2 \approx m_\pi^2$, the 7% charge splitting of m_π^2 will lead to only about a 3% charge-dependent effect from the propagator. Thus, if a 6% charge-splitting of g_π^2 is used, this will not only override the pion-mass effect, it will reverse it.

Based upon this argument and on our numerical results, one can then estimate that a charge-splitting of g_π^2 of only about 3% (e. g., $g_{\pi^0}^2/4\pi = 13.6$ and $g_{\pi^\pm}^2/4\pi = 14.0$) would erase all CIB prediction of the singlet scattering length that is based upon the conventional mechanism of pion mass splitting.

4 Conclusions

All current determinations of the neutral-pion coupling constant seem to agree on a 'low' value, like $g_{\pi^0}^2/4\pi = 13.6 \pm 0.3$. However, for the charged-pion coupling constant, there is no such agreement. While some recent determinations of $g_{\pi^\pm}^2/4\pi$ come up with a value close to $g_{\pi^0}^2/4\pi$, the Uppsala group [18] obtains $g_{\pi^\pm}^2/4\pi = 14.52 \pm 0.26$ which implies a large charge-dependence of g_π.

In this paper, we have investigated the consequences of such a large charge-dependence of g_π for the conventional explanation of the charge-dependence of the $^1S_0$ scattering length, a_s. We find that a charge-splitting of the coupling constant, defined by $\Delta g_\pi^2/4\pi \equiv (g_{\pi^\pm}^2 - g_{\pi^0}^2)/4\pi$, of $\Delta g_\pi^2/4\pi = 0.4$ would wipe out the effect of the conventional mechanism (namely, pion mass splitting) and a splitting of $\Delta g_\pi^2/4\pi = 0.8$ would even reverse the charge-dependence of a_s [29].

Besides pion mass splitting, we do not know of any other essential mechanism to explain the charge-dependence of a_s. Therefore, it is unlikely that this mechnism is annihilated by a charge-splitting of g_π. This may be taken as an indication that there is no significant charge splitting of the πNN coupling constant.

Consequently, charge-dependence of g_π is most likely not the resolution of the large differences in recent g_π determinations; which implies that we are dealing here with true discrepancies. The reasons for these discrepancies may be large (unknown) sytematic errors and/or a gross underestimation of the errors in essentially all present g_π determinations.

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$$\frac{g_{\pi^0 pp}^2}{4\pi} = \left(\frac{2M_p}{m_{\pi^\pm}}\right)^2 f_{\pi^0 pp}^2 = 180.773 f_{\pi^0 pp}^2 \qquad (18)$$

and

$$\frac{g^2_{\pi^\pm}}{4\pi} = \left(\frac{M_p + M_n}{m_{\pi^\pm}}\right)^2 f^2_{\pi^\pm} = 181.022 f^2_{\pi^\pm} \ . \tag{19}$$

with $M_p = 938.272$ MeV the proton mass, $M_n = 939.566$ MeV the neutron mass, and $m_{\pi^\pm} = 139.570$ MeV the mass of the charged pion.

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